

Vector double cross product:
 $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

Characteristic plasma parameters:

$$\lambda_{dB} = \sqrt{\frac{\epsilon_0 k_B T_e}{n_e e^2}} = 743 \sqrt{\frac{T_e(\text{eV})}{n_e(\text{cm}^{-3})}} \text{ (cm)} ; \quad \nu_{pe} = \frac{\omega_{pe}}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{e^2 n_e}{m_e \epsilon_0}} = 8.98 \sqrt{n_e(\text{cm}^{-3})} \text{ (kHz)}$$

$$\nu_{ce}(\text{GHz}) = 28 B(\text{T}) ; \quad \nu_{ee}(\text{s}^{-1}) = 4 \cdot 10^{-12} \frac{n_e(\text{m}^{-3}) \ln \Lambda}{T_e^{3/2}(\text{eV})}$$

Guiding center drifts:

$$\mathbf{v}_{gc,E \times B} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} ; \quad \mathbf{v}_{gc,\nabla B} = \frac{mv_\perp^2}{2qB^3} \mathbf{B} \times \nabla B ; \quad \mathbf{v}_{gc,cB} = \frac{mv_\parallel^2}{qB^3} \mathbf{B} \times \nabla B$$

Boltzmann equation: $\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \frac{\partial f_\alpha}{\partial \mathbf{r}} + \frac{q_\alpha}{m_\alpha} [\mathbf{E} + \mathbf{v} \times \mathbf{B}] \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = \left(\frac{\partial f_\alpha}{\partial t} \right)_c$

Electro- and magnetostatics:

$$\epsilon_0 \oint_S \mathbf{E} \cdot \mathbf{n} d\sigma = Q_{encl} ; \quad \oint_C \mathbf{B} \cdot \mathbf{t} ds = \mu_0 I_{encl}$$

Maxwell's equations:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} ; \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} ; \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} ; \quad \nabla \cdot \mathbf{B} = 0$$

with $\rho = \sum_\alpha q_\alpha \int f_\alpha(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}$ and $\mathbf{j} = \sum_\alpha q_\alpha \int \mathbf{v} f_\alpha(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}$.

Single-fluid MHD equations:

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 ; \quad \rho \frac{du}{dt} = \rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \mathbf{j} \times \mathbf{B} - \nabla p ; \quad \frac{d}{dt}(p \rho^{-\gamma}) = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} ; \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{j} ; \quad \mathbf{E} + \mathbf{u} \times \mathbf{B} = -\mathbf{j}$$

Two-fluid MHD equations:

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{u}_s) = 0$$

$$m_s n_s \left[\frac{\partial \mathbf{u}_s}{\partial t} + (\mathbf{u}_s \cdot \nabla) \mathbf{u}_s \right] = q_s n_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) - \nabla \cdot \mathbf{P}_s - m_s n_s \sum_t \nu_{st} (\mathbf{u}_s - \mathbf{u}_t)$$

For the plasma with cylindrical symmetry, we have pressure profile $p(r)$ and axial magnetic field $B_z(r)$. The total vacuum magnetic field B_0 for $r < r < b$ is uniform.

To solve the two-dimensional equilibrium MHD force balance equation, we need to calculate the following terms:
 $\nabla \left(\rho + \frac{B^2}{2\mu_0} \right) = \frac{1}{\rho} (\mathbf{B} \cdot \nabla) \mathbf{B}$
 and give an interpretation of each term.

- Apply the above MHD force balance equation to the theta pitch to obtain an equation for $n(r)$, $B(r)$ and $\mathbf{u}(r)$.
- Assume that the total induced current in the plasma is J_p . Calculate the normalization constant A in the expression $J_p(r) = Ar(a-r)$.
- Calculate the magnetic field inside the plasma $B_r(r)$.